

ON BONNESEN–STYLE ISOPERIMETRIC INEQUALITIES FOR n -SIMPLICES

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Abstract. In this paper, we derive some Bonnesen-style inequalities referring to n -dimensional simplices in the spirit of isoperimetric problems. These inequalities combine various metric quantities of simplices and yield, in several cases, characterizations of regular simplices. Also related reverse Bonnesen-style inequalities are derived.

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