

## JACKSON–STECHKIN TYPE INEQUALITY IN WEIGHTED LORENTZ SPACES

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*Abstract.* In the present work we consider the modulus of smoothness, defined by means of the Steklov operator in weighted Lorentz spaces and prove the Jackson-Stechkin type direct theorem of trigonometric approximation. In the particular case we obtain a result on the constructive characterization of the generalized Lipschitz classes defined in these spaces. Simultaneous approximation of functions is also considered.

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