A NEW ESTIMATE OF THE DIFFERENCE AMONG QUASI–ARITHMETIC MEANS

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Abstract. In the 1960s Cargo and Shisha proved some majorizations for the distance among quasi-arithmetic means (defined as $f^{-1}(\sum_{i=1}^{n} w_i f(a_i))$ for any continuous, strictly monotone function $f : I \to \mathbb{R}$, where $I$ is an interval, and $(a_1, \ldots, a_n)$ is a vector with entries in $I$, $(w_1, \ldots, w_n)$ is a sequence of corresponding weights $w_i > 0$, $\sum w_i = 1$).

Nearly thirty years later, in 1991, Páles presented an iff condition for a sequence of quasi-arithmetic means to converge to another QA mean. It was closely related with the three-parameter operator $(f(x) - f(y))/(f(x) - f(z))$.

The author presented recently an estimate for the distance among such quasi-arithmetic means whose underlying functions satisfy some smoothness conditions. Used was the operator $f \mapsto f''/f'$ introduced in the 1940s by Mikusiński and Łojasiewicz. It is natural to look for similar estimate(s) in the case of the underlying functions not being smooth. For instance, by the way of using Páles’ operator. This is done in the present note. Moreover, the result strengthens author’s earlier estimates.


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REFERENCES