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A NEW ESTIMATE OF THE DIFFERENCE AMONG QUASI–ARITHMETIC MEANS

PAWEŁ PASTECZKA

Abstract. In the 1960s Cargo and Shisha proved some majorizations for the distance among quasi-arithmetic means (defined as $f^{-1}\left(\sum_{i=1}^{n} w_i f(a_i)\right)$ for any continuous, strictly monotone function $f\colon I \to \mathbb{R}$, where I is an interval, and $(a_1,\ldots,a_n)$ is a vector with entries in I, $(w_1,\ldots,w_n)$ is a sequence of corresponding weights $w_i > 0$, $\sum w_i = 1$).

Nearly thirty years later, in 1991, Páles presented an iff condition for a sequence of quasi-arithmetic means to converge to another QA mean. It was closely related with the three-parameter operator $(f(x)-f(y))/(f(x)-f(z))$.

The author presented recently an estimate for the distance among such quasi-arithmetic means whose underlying functions satisfy some smoothness conditions. Used was the operator $f \mapsto f''/f'$ introduced in the 1940s by Mikusiński and Łojasiewicz. It is natural to look for similar estimate(s) in the case of the underlying functions *not* being smooth. For instance, by the way of using Páles' operator. This is done in the present note. Moreover, the result strengthens author's earlier estimates.

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