SHARP FORM OF MODIFIED WEIGHTED
HARDY INEQUALITIES OF TRACE TYPE

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Abstract. The extremal function and the best constant for a modified weighted Hardy inequality of trace type are investigated by using the Bliss' argument. Computational difficulties of the self-contained proof are overcome by the techniques taken from a paper by Talenti.

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