

SOME RESULTS FOR HAUSDORFF OPERATORS

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Abstract. In this paper, we give the sufficient conditions for the boundedness of the (fractional) Hausdorff operators on the Lebesgue spaces with power weights. In some special cases, these conditions are the same and also necessary. As an application, we obtain a better lower bound of fractional Hardy operators on the Lebesgue spaces compared with a result of the paper [25].

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