

WEIGHTED APPROXIMATION BY BASKAKOV OPERATORS

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Abstract. The weighted approximation errors of Baskakov operator is characterized for weights of the form $w(x) = x^{\gamma_0} (1+x)^{\gamma_\infty}$, where $\gamma_0 \in [-1, 0]$, $\gamma_\infty \in \mathbb{R}$. Direct inequalities and strong converse inequalities of type A are proved in terms of the weighted K -functional.

Mathematics subject classification (2010): 41A36, 41A25, 41A27, 41A17.

Keywords and phrases: Baskakov operator, K -functional, direct theorem, strong converse theorem.

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