JACKSON KERNELS: A TOOL FOR ANALYSING THE DECAY OF EIGENVALUE SEQUENCES OF INTEGRAL OPERATORS ON THE SPHERE

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Abstract. Decay rates for the sequence of eigenvalues of positive and compact integral operators have been largely investigated for a long time in the literature. In this paper, the focus will be on positive integral operators acting on square integrable functions on the unit sphere and generated by a kernel satisfying a Hölder type assumption defined by average operators. In the approach to be presented here, the decay rate will be reached from convenient estimations on the eigenvalues of the operator themselves, with the help of specific properties of a generic approximation operator defined through the so-called generalized Jackson kernels. The decay rate has the same structure of those known to hold in the cases in which the Hölder condition is the classical one. Therefore, within the spherical setting, the abstract approach to be introduced here extends some classical results on the topic.

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