ON CRITICAL CONDITION FOR A WEIGHTED INTEGRAL SYSTEM WITH NEGATIVE EXPONENTS

LILI HUANG

Abstract. This paper is concerned with the integral system
\[
\begin{aligned}
&u(x) = \int_{\mathbb{R}^n} \frac{|x|^{\alpha}|y|^{\beta}|x-y|^s}{v^q(y)}dy, \quad u > 0 \text{ in } \mathbb{R}^n, \\
v(x) = \int_{\mathbb{R}^n} \frac{|x|^{\beta}|y|^{\alpha}|x-y|^s}{u^p(y)}dy, \quad v > 0 \text{ in } \mathbb{R}^n,
\end{aligned}
\]
where \( n \geq 1, \alpha, \beta, s > 0 \) and \( p, q < 0 \). Such an integral system appears in the study of the conformal geometry and the weighted Hardy-Littlewood-Sobolev inequality. We obtain that
\[
\frac{1}{p+1} + \frac{1}{q+1} = -\frac{\alpha + \beta + s}{n},
\]
is a necessary condition for the existence of the \( C^1 \) positive entire solutions, which is also the necessary and sufficient condition for the invariant of the system and some energy functionals under the scaling transformation.


Keywords and phrases: Singular integral equation, critical condition, negative exponent.

REFERENCES


