# JOINTLY SUBADDITIVE MAPPINGS INDUCED
# BY OPERATOR CONVEX FUNCTIONS

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*Abstract.* In this paper, we study jointly subadditive mappings induced by operator convex functions and generalized inverses of positive linear maps. We formulate conditions under which the inequalities $TfT^-\left(\sum_{k=1}^{n} T_k A_k\right) \leqslant \sum_{k=1}^{n} T_k f(A_k)$ and $TfT^-\Phi(A) \leqslant \Phi(f(A))$ hold, where $f$ is an operator convex function, $A, A_k \in \mathbb{B}(H)$ with Hilbert space $H$, and $T$, $T_k$ and $\Phi$ are positive linear maps (not necessarily unital) on $\mathbb{B}(H)$, with a (reflexive) generalized inverse $T^-$ of $T$. We also show that the transformation $TfT^-(B)$ is jointly subadditive in $(T, B)$ and antimonotone in $T(I)$.

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