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SHARP INEQUALITIES FOR HILBERT TRANSFORM IN A VECTOR–VALUED SETTING

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Abstract. The paper is devoted to the study of the periodic Hilbert transform $\mathscr{H}$ in the vector valued setting. Precisely, for any positive integer N we determine the norm of $\mathscr{H}$ as an operator from $L^\infty(\mathbb{T};\ell_\infty^N)$ to $L^p(\mathbb{T};\ell_\infty^N)$, $1 \leqslant p < \infty$, and from $L^p(\mathbb{T};\ell_1^N)$ to $L^1(\mathbb{T};\ell_1^N)$, for $1 < p \leqslant \infty$. The proof rests on the existence of a certain family of special harmonic functions.

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