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WEAK MONOTONICITY AND CHEBYSHEV TYPE INEQUALITY

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Abstract. The weak monotonic function is defined in this paper. We will study the relationship between the weak monotonic function and the Schur-function. We show that a Schur-convex function is a weak increasing function under the proper hypotheses. By means of the theory of weak monotonic function with appropriate assumptions, we have established a Chebyshev type inequality as follows:

$$\frac{\langle \mathbf{a}, \mathbf{b} \rangle}{\langle \mathbf{a}^*, \mathbf{b}^* \rangle} \geqslant \frac{\|\mathbf{a}\|_p}{\|\mathbf{a}^*\|_p} \cdot \frac{\|\mathbf{b}\|_q}{\|\mathbf{b}^*\|_q}.$$

As the application of the inequality, a new proof of Marshall's inequality is obtained.

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