Mathematical Inequalities & Applications
Volume 18, Number 1 (2015), 233–240
doi:10.7153/mia-18-17

THE BEST BOUND FOR n–DIMENSIONAL FRACTIONAL HARDY OPERATORS

FAYOU ZHAO AND SHANZHEN LU

Abstract. In this note, we precisely evaluate the operator norm of the fractional Hardy operator $\mathbb{H}_\beta$ from $L^p(\mathbb{R}^n)$ to $L^q(\mathbb{R}^n)$, where $0<\beta<n$, $1<p<q<\infty$ and $1/p-1/q=\beta/n$. By this we extend the result of Bliss [1] to the case of high dimension and improve our result in [7].

Mathematics subject classification (2010): Primary 26D10, 26D15, 42B99.
Keywords and phrases: Best bound, the n-dimensional fractional Hardy operator.

REFERENCES

[1] G. A. BLISS, *An integral inequality*, J. Lond. Math. Soc., **5**, (1930), 40–46.
[2] V. I. BURENKOV, P. JAIN, T. V. TARARYKOVA, *On boundedness of the Hardy operator in Morrey-type spaces*, Eurasian Math. J., **2**, (2011), 52–80.
[3] D. W. BOYD, *Inequalities for positive integral operators*, Pac. J. Math., **38**, (1971), 9–24.
[4] Z. W. FU, Z. G. LIU, S. Z. LU AND H. B. WANG, *Characterization for commutators of n-dimensional fractional Hardy operators*, Sci. China (Ser. A)., **50**, (2007), 1418–1426.
[5] Z. W. FU, L. GRAFAKOS, S. Z. LU AND F. Y. ZHAO, *Sharp bounds for m-linear Hardy and Hilbert operators*, Houston J. Math., **38**, (2012), 225–244.
[6] B. OPIC AND A. KUFNER, *Hardy-type inequalities*, Pitman Research Notes in Mathematics Series, vol. **219**, Longman Group UK Limited, London, 1990.
[7] S. Z. LU, D. Y. YAN AND F. Y. ZHAO, *Sharp bounds for Hardy type operators on higher-dimensional product spaces*, J. Inequal. Appl., **2013**, 1 (2013), 1–11.
[8] V. M. MANAKOV, *On the best constant in weighted inequalities for Riemann-Liouville integrals*, Bull. London Math. Soc., **24**, 5 (1992), 442–448.