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CHARACTERIZATION OF THE RESTRICTED TYPE SPACES $R(X)$

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Abstract. We study functorial properties of the spaces $R(X)$, which have been recently introduced as a central tool in the analysis of the Hardy operator minus the identity on decreasing functions. In particular, we provide conditions on a minimal Lorentz space Λ_φ so that the equation $R(X) = \Lambda_\varphi$ has a solution within the category of rearrangement invariant (r.i.) spaces. Moreover, we show that if $R(X) = \Lambda_\varphi$, then we can always take X to be the minimal r.i. Banach range space for the Hardy operator defined in Λ_φ.

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