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SHARP ESTIMATES REGARDING THE REMAINDER OF THE ALTERNATING HARMONIC SERIES

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Abstract. In the present paper we obtain enhanced estimates regarding the remainder of the alternating harmonic series. More precisely, we show that

$$\frac{1}{4n^2+a} < \left| \sum_{k=1}^{n} (-1)^{k-1}\frac{1}{k} - (-1)^{n-1}\frac{1}{2n} - \ln 2 \right| \leqslant \frac{1}{4n^2+b},$$

for all $n \in \mathbb{N}$, with $a = 2$ and $b = \frac{2(3-4\ln 2)}{2\ln 2-1} = 1.177398899\ldots$. In addition, the constants a and b are the best possible with the above-mentioned property.

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REFERENCES

[1] H. ALZER, *Inequalities for the gamma and polygamma functions*, Abh. Math. Sem. Univ. Hamburg **68** (1998), 363–372
[2] C.-P. CHEN, *Inequalities for the Euler–Mascheroni constant*, Appl. Math. Lett. **23**, 2 (2010), 161–164.
[3] C.-P. CHEN AND F. QI, *The best lower and upper bounds of harmonic sequence*, RGMIA **6**, 2 (2003), 303–308.
[4] C.-P. CHEN AND C. MORTICI, *New sequence converging towards the Euler–Mascheroni constant*, Comput. Math. Appl. **64**, 4 (2012), 391–398.
[5] C.-P. CHEN AND H. M. SRIVASTAVA, *New representations for the Lugo and Euler–Mascheroni constants*, Appl. Math. Lett. **24**, 7 (2011), 1239–1244.
[6] O. FURDUI, *Limits, Series, and Fractional Part Integrals. Problems in Mathematical Analysis*, Springer, New York (2013).
[7] I. S. GRADSHTEYN AND I. M. RYZHIK, *Table of Integrals, Series, and Products* (*7th ed.*), Elsevier/Academic Press, Amsterdam (2007).
[8] B.-N. GUO AND F. QI, *Sharp bounds for harmonic numbers*, Appl. Math. Comput. **218**, 3 (2011), 991–995.
[9] D. K. KAZARINOFF, *A simple derivation of the Leibniz-Gregory series for* $\pi/4$, Amer. Math. Monthly **62**, 10 (1955), 726–727.
[10] F. W. J. OLVER (ED.), D. W. LOZIER (ED.), R. F. BOISVERT (ED.) AND C. W. CLARK (ED.), *NIST Handbook of Mathematical Functions*, Cambridge University Press, Cambridge, 2010.
[11] W. RAUTENBERG, *Zur Approximation von e durch* $\left(1+\frac{1}{n}\right)^n$ (*On the approximation of e by* $\left(1+\frac{1}{n}\right)^n$), Math. Semesterber. **33** (1986), 227–236.
[12] A. SÎNTĂMĂRIAN, *A new proof for estimating the remainder of the alternating harmonic series*, Creat. Math. Inform. **21**, 2 (2012), 221–225.
[13] L. TÓTH, *Problem E3432*, Amer. Math. Monthly **98**, 3 (1991), 264.
[14] L. TÓTH, *Problem E3432* (*Solution*), Amer. Math. Monthly **99**, 7 (1992), 684–685.
[15] L. TÓTH, *On a class of Leibniz series*, Rev. Anal. Numér. Théor. Approx. **21**, 2 (1992), 195–199.
[16] L. TÓTH AND J. BUKOR, *On the alternating series* $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$, J. Math. Anal. Appl. **282**, 1 (2003), 21–25.