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BOUNDEDNESS FOR RIESZ–TYPE POTENTIAL OPERATORS ON HERZ–MORREY SPACES WITH VARIABLE EXPONENT

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Abstract. In this paper, the Riesz-type potential operator of variable order $\beta(x)$ is shown to be bounded from the Herz-Morrey spaces $M\dot{K}^{\alpha,\lambda}_{p_1,q_1(\cdot)}(\mathbb{R}^n)$ with variable exponent $q_1(x)$ into the weighted space $M\dot{K}^{\alpha,\lambda}_{p_2,q_2(\cdot)}(\mathbb{R}^n,\omega)$, where $\omega=(1+|x|)^{-\gamma(x)}$ with some $\gamma(x)>0$ and $1/q_1(x)-1/q_2(x)=\beta(x)/n$ when $q_1(x)$ is not necessarily constant at infinity. It is assumed that the exponent $q_1(x)$ satisfies the logarithmic continuity condition both locally and at infinity and $1<q_1(\infty)\leqslant q_1(x)\leqslant(q_1)_+<\infty\,(x\in\mathbb{R}^n)$.

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REFERENCES

[1] O. KOVÁČIK AND J. RÁKOSNÍK, *On spaces $L^{p(x)}$ and $W^{k,p(x)}$*, Czechoslovak Mathematical Journal, **41**, 4 (1991), 592–618.
[2] D. CRUZ-URIBE, L. DIENING AND A. FIORENZA, *A new proof of the boundedness of maximal operators on variable Lebesgue spaces*, Bollettino della Unione Matematica Italiana, **2**, 1 (2009), 151–173.
[3] D. CRUZ-URIBE, A. FIORENZA, J. MARTELL AND C. PÉREZ, *The boundedness of classical operators on variable L^p spaces*, Ann. Acad. Sci. Fenn. Math., **31** (2006), 239–264.
[4] D. CRUZ-URIBE, A. FIORENZA AND C. NEUGEBAUER, *The maximal function on variable L^p spaces*, Ann. Acad. Sci. Fenn. Math., **28** (2003), 223–238.
[5] L. DIENING, P. HARJULEHTO, P. HÄSTÖ AND M. RUZICKA, *Lebesgue and Sobolev Spaces with Variable Exponents*, Springer-Verlag, Lecture Notes in Mathematics, Vol. **2017**, Berlin,2011.
[6] D. CRUZ-URIBE AND A. FIORENZA, *Variable Lebesgue Spaces: foundations and harmonic analysis, Birkhäuser/Springer*, Applied and Numerical Harmonic Analysis, Heidelberg, 2013.
[7] L. DIENING, *Maximal functions on Musielak-Orlicz spaces and generalized Lebesgue spaces*, Bull. Sci. Math., **129** (2005), 657–700.
[8] L. DIENING, *Maximal functions on generalized Lebesgue spaces $L^{p(\cdot)}$*, Mathematical Inequalities and Applications, **7**, 2 (2004), 245–253.
[9] L. DIENING, P. HARJULEHTO, P. HÄSTÖ, Y. MIZUTA AND T. SHIMOMURA, *Maximal functions in variable exponent spaces: limiting cases of the exponent*, Ann. Acad. Sci. Fenn. Math., **34** (2009), 503–522.
[10] T. KOPALIANI, *Infimal convolution and Muckenhoupt $A_{p(\cdot)}$ condition in variable L^p spaces*, Arch. Math., **89**, 2 (2007), 185–192.
[11] A. LERNER, *On some questions related to the maximal operator on variable L^p spaces*, Trans. Amer. Math. Soc., **362** (2010), 4229–4242.
[12] A. NEKVINDA, *Hardy-Littlewood maximal operator on $L^{p(x)}(\mathbb{R}^n)$*, Math. Inequal. Appl., **7**, 2 (2004), 255–265.
[13] L. PICK AND M. RUŽIČKA, *An example of a space $L^{p(\cdot)}$ on which the Hardy-Littlewood maximal operator is not bounded*, Expo. Math., **19** (2001), 369–371.

Paper MIA-18-35

[14] A. ALMEIDA AND D. DRIHEM, *Maximal, potential and singular type operators on Herz spaces with variable exponents*, Journal of Mathematical Analysis and Applications, **394**, 2 (2012), 781–795.
[15] S. SAMKO, *Variable exponent Herz spaces*, Mediterranean Journal of Mathematics, **10**, 4 (2013), 2005–2023.
[16] S. SAMKO, *Erratum to "Variable exponent Herz spaces"*, Mediterr. J. Math. DOI: 10.1007/s00009-013-0285-X,2013, Mediterranean Journal of Mathematics, **10**, 4 (2013), 2027–2030.
[17] A. ALMEIDA, J. HASANOV AND S. SAMKO, *Maximal and potential operators in variable exponent Morrey spaces*, Georgian Mathematical Journal, **15**, 2 (2008), 195–208.
[18] M. IZUKI, *Boundedness of sublinear operators on Herz spaces with variable exponent and application to wavelet characterization*, Analysis Mathematica, **36** (2010), 33–50.
[19] J. L. WU, *Boundedness of some sublinear operators on Herz-Morrey spaces with variable exponent*, Georgian Mathematical Journal, **21**, 1 (2014), 101–111.
[20] V. GULIYEV, J. HASANOV AND S. SAMKO, *Boundedness of the maximal, potential and singular operators in the generalized variable exponent Morrey spaces*, Mathematica Scandinavica, **107**, 2 (2010), 285–304.
[21] V. GULIYEV, J. HASANOV AND S. SAMKO, *Maximal, potential and singular operators in the local "complementary" variable exponent Morrey type spaces*, Journal of Mathematical Analysis and Applications, **401**, 1 (2013), 72–84.
[22] M. IZUKI, *Fractional integrals on Herz-Morrey spaces with variavle exponent*, Hiroshima Math J, **40** (2010), 343–355.
[23] S. SAMKO, *Convolution and potential type operators in* $L^{p(x)}$, Integral Transforms and Special Functions, **7**, 3–4 (1998), 261–284.
[24] P. HÄSTÖ, *Local-to-global results in variable exponent spaces*, Mathematical Research Letters, **16**, 2 (2009), 263–278.
[25] V. GULIYEV, J. HASANOV AND S. SAMKO, *Boundedness of maximal, potential type, and singular integral operators in the generalized variable exponent Morrey type spaces*, Journal of Mathematical Sciences, **170**, 4 (2010), 423–443.
[26] L. DIENING, *Riesz potential and Sobolev embeddings on generalized Lebesgue spaces and Sobolev spaces* $L^{p(\cdot)}$ *and* $W^{k,p(\cdot)}$, Mathematische Nachrichten, **268** (2004), 31–43.
[27] V. KOKILASHVILI AND S. SAMKO, *On Sobolev theorem for Riesz type potentials in the Lebesgue spaces with variable exponent*, Zeitschrift für Analysis und ihre Anwendungen(Journal for Analysis and its Applications), **22**, 4 (2003), 899–910.
[28] V. GULIYEV AND S. SAMKO, *Maximal, Potential and singular operators in the generalized variable exponent Morrey spaces on unbounded sets*, Journal of Mathematical Sciences, **193**, 2 (2013), 228–248.
[29] P. ZHANG AND J. L. WU, *Boundedness of commutators of the fractional Hardy operators on Herz-Morrey spaces with variable exponent*, Advances in Mathematics (China), In Press.