

PROPERTIES OF MODIFIED BESSEL FUNCTIONS AND COMPLETELY MONOTONIC DEGREES OF DIFFERENCES BETWEEN EXPONENTIAL AND TRIGAMMA FUNCTIONS

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Abstract. In the paper, the author establishes inequalities, monotonicity, convexity, and unimodality for functions concerning the modified Bessel functions of the first kind and compute the completely monotonic degrees of differences between the exponential and trigamma functions.

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