Mathematical
Inequalities
& Applications
Volume 18, Number 2 (2015), 519–527
doi:10.7153/mia-18-38

ON THE BOUNDEDNESS OF FRACTIONAL TYPE MARCINKIEWICZ INTEGRAL OPERATORS

QINGYING XUE, KÔZÔ YABUTA AND JINGQUAN YAN

Abstract. We show that a broad family of fractional type Marcinkiewicz integral operators with the kernel belonging to $L^1(S^{n-1})$ is bounded from the Triebel-Lizorkin space $F^{\alpha}_{pq}(\mathbb{R}^n)$ to Lebesgue space $L^p(\mathbb{R}^n)$, which improves some known results significantly. This is done by exploiting a local but more general fractional version of Littlewood-Paley g-function.

Mathematics subject classification (2010): Primary 42B20; Secondary 42B25, 47G10.
Keywords and phrases: L^p boundedness; Marcinkiewicz integral; fractional integral operator; Triebel-Lizorkin spaces.

REFERENCES

[1] A. AL-SALMAN, *A class of Marcinkiewicz type integral operators*, Comm. Math. Anal. **13** No. 2 (2012), 56–81.
[2] A. AL-SALMAN, H. AL-QASSEM, L. CHENG AND Y. PAN, *L^p bounds for the function of Marcinkiewicz*, Math. Res. Lett. **9** (2002), 697–700.
[3] A. BENEDEK, A. CALDERÓN, R. PANZONE, *Convolution operators on Banach space valued functions*, Proc. Natl. Acad. Sci. **48** (1962), 356–365.
[4] J. CHEN, D. FAN AND Y. YING, *Singular integral operators on function spaces*, J. Math. Anal. Appl. **276** (2002), 691–708.
[5] Y. DING, D. FAN AND Y. PAN, *L^p-boundedness of Marcinkiewicz integrals with Hardy space function kernels*, Acta Math. Sinica (English Ser.) **16** (2000), 593–600.
[6] Y. DING, D. FAN AND Y. PAN, *On the L^p boundedness of Marcinkiewicz Integrals*, Michigan Math. J. **50** (2002), 17–26.
[7] Y. DING, C.-C. LIN AND S. SHAO, *On Marcinkiewicz integral with variable kernels*, Indiana Univ. Math. J. **53** (2004), 805–822.
[8] J. DUOANDIKOETXEA AND J. L. RUBIO DE FRANCIA, *Maximal and singular integral operators via Fourier transform estimates*, Invent. Math. **84** (1986), 541–561.
[9] L. HÖRMANDER, *Estimates for translation invariant operators in L^p spaces*, Acta Math. **104** (1960), 93–140.
[10] J. MARCINKIEWICZ, *Sur quelques integrales de type de Dini, Annales de la Sociéeté Polonaise*, **17** (1938), 42–50.
[11] M. SAKAMOTO, K. YABUTA, *Boundedness of Marcinkiewicz functions*, Studia Math. **135** (1999), 103–142.
[12] E. M. STEIN, *On the function of Littlewood-Paley, Lusin and Marcinkiewicz*, Trans. Amer. Math. Soc. **88** (1958), 430–466.
[13] H. TRIEBEL, *Theory of Function Spaces*, Birkhäuser (1983).
[14] T. WALSH, *On the function of Marcinkiewicz*, Studia Math. **44** (1972), 203–217.
[15] Q. XUE, K. YABUTA, J. YAN, *Fractional type Marcinkiewicz integral operators on function spaces*, Forum Mathematicum, DOI: 10.1515/forum-2013-0200, to appear.
[16] A. ZYGMUND, *On certain integrals*, Trans. Amer. Math. Soc. **55** (1944), 170–204.

Paper MIA-18-38