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NEW INEQUALITIES AND INFINITE PRODUCT FORMULAS FOR TRIGONOMETRIC AND THE LEMNISCATE FUNCTIONS

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Abstract. In this paper, we give a new approach to prove inequalities for the Schwab-Borchardt mean, the lemniscatic mean and the arithmetic geometric mean. Additionally, we apply these means to inequalities for trigonometric functions or the lemniscate functions by considering several functional inequalities. One of these applications includes infinite product formulas for the lemniscate function and the arithmetic geometric mean by considering several functional equations.

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