Mathematical
Inequalities
& Applications
Volume 18, Number 2 (2015), 541–553
doi:10.7153/mia-18-40

A YOUNG–LIKE INEQUALITY WITH APPLICATIONS TO THE COMMUTATOR ESTIMATES

JIANG XU

Abstract. In the recent decade, Fourier analysis techniques based on the Littlewood-Paley decomposition have proved to be very efficient in the study of PDEs, since J.-M. Bony introduced the paradifferential calculus. Of those techniques, commutator estimates play the crucial role in dealing with bilinear estimates. In this paper, we develop a Young-like inequality which is the generalization of the classical Young's convolution inequality. The new inequality enables us to obtain various commutator estimates in the L^p-framework, which creates the basis to obtain different nonlinear a priori estimates in the analysis of PDEs.

Mathematics subject classification (2010): 35Q30, 76D05, 35Q35, 42B35.
Keywords and phrases: Young's inequality, Littlewood-Paley decomposition, commutator estimates, Bony's paradifferential calculus.

REFERENCES

[1] H. ABIDI, *Equation de Navier-Stokes avec densité et viscosité variables dans l'espace critique*, Revista Math. Iber., **23**, 2 (2007), 537–586.
[2] H. ABIDI, G. L. GUI AND P. ZHANG, *On the wellposedness of three-dimensional inhomogeneous Navier-Stokes equations in the critical spaces*, Arch. Ration. Mech. Anal., **204**, 1 (2012), 189–230.
[3] H. BAHOURI, J. Y. CHEMIN AND R. DANCHIN, *Fourier Analysis and Nonlinear Partial Differential Equations*, Grundlehren der mathematischen Wissenschaften, Berlin: Springer-Verlag, 2011.
[4] J.-Y. CHEMIN AND N. LERNER, *Flot de champs de vecteurs non lipschitziens et équations de Navier-Stokes*, J. Differential Equations, **121**, 2 (1995), 314–328.
[5] Q. L. CHEN, C. X. MIAO AND Z. F. ZHANG, *Global well-posedness for the compressible Navier-Stokes equations with the highly oscillating initial velocity*, Comm. Pure Appl. Math., **63**, 9 (2010), 1173–1224.
[6] Q. L. CHEN, C. X. MIAO AND Z. F. ZHANG, *Well-posedness in critical spaces for the compressible Navier-Stokes equations with density dependent viscosities*, Rev. Mat. Iberoamericana, **26**, 3 (2010), 915–946.
[7] R. DANCHIN, *Global existence in critical spaces for compressible Navier-Stokes equations*, Inventiones Mathematicae, **141**, 3 (2000), 579–614.
[8] R. DANCHIN, *Global existence in critical spaces for flows of compressible viscous and heat-conductive gases*, Arch. Ration. Mech. Anal., **160**, 1 (2001), 1–39.
[9] R. DANCHIN, *On the well-posedness of the incompressible density-dependent Euler equations in the L^p framework*, J. Differential Equations, **248**, 8 (2010), 2130–2170.
[10] R. DANCHIN AND F. FANELLI, *The well-posedness issue for the density-dependent Euler equations in endpoint Besov spaces*, J. Math. Pures Appl., **96**, 3 (2011), 253–278.
[11] L. GRAFAKOS, *Classical Fourier Analysis* (2nd), New York: Springer, 2008.
[12] C. C. HAO AND H. L. LI, *Well-posedness for a multidimensional viscous liquid-gas two phase flow model*, SIAM J. Math. Anal., **44**, 3 (2012), 1304–1332.
[13] M. PAICU AND P. ZHANG, *Global solutions to the 3-D incompressible anisotropic Navier-Stokes system in the critical spaces*, Commun. Math. Phys., **307**, 3 (2011), 713–759.
[14] M. PAICU AND P. ZHANG, *Global solutions to the 3-D incompressible inhomogeneous Navier-Stokes system*, J. Funct. Anal., **262**, 8 (2012), 3556–3584.

Paper MIA-18-40

[15] J. Z. QIAN AND Z. F. ZHANG, *Global well-posedness for compressible viscoelastic fluids near equilibrium*, Arch. Ration. Mech. Anal., **198**, 3 (2010), 835–868.
[16] J. XU, *Relaxation-time limit in the isothermal hydrodynamic model for semiconductors*, SIAM J. Math. Anal., **40**, 5 (2008), 1979–1991.
[17] J. XU, *Global classical solutions to the compressible Euler-Maxwell equations*, SIAM J. Math. Anal., **43**, 6 (2011), 2688–2718.
[18] J. XU AND Z. J. WANG, *Relaxation limit in Besov spaces for compressible Euler equations*, J. Math. Pures Appl., **99**, 1 (2013), 43–61.
[19] T. ZHANG, *Global wellposed problem for the 3-D incompressible anisotropic Navier-Stokes equations in an anisotropic space*, Commun. Math. Phys., **287**, 1 (2009), 211–224.
[20] T. ZHANG AND D. Y. FANG, *Global existence of strong solution for equations related to the incompressible viscoelastic fluids in the critical L^p framework*, SIAM J. Math. Anal., **44**, 4 (2012), 2266–2288.
[21] T. ZHANG AND D. Y. FANG, *Global wellposed problem for the 3-D incompressible anisotropic Navier-Stokes equations*, J. Math. Pures Appl., **90**, 5 (2008), 413–449.