Mathematical
Inequalities
& Applications
Volume 18, Number 2 (2015), 581–588 doi:10.7153/mia-18-43

CONCAVITY OF THE FUNCTION $f(A) = \det(I-A)$ FOR DENSITY OPERATOR A

YAN-NI DOU, HAI DU AND YUAN LI

Abstract. According to the Minkowski determinant theorem the function $f(A) = \det(I-A)^{1/n}$ is concave on the set of $n \times n$ positive contractive matrices, that is, $0 \leqslant A \leqslant I$. When $n > 1$ the exponent $1/n$ can not be removed. On the other hand $\det(I-A)$ has meaning even for a trace-class Hilbert space operator A. In this paper we will prove that the function $f(A) = \det(I-A)$ is concave on the set of density operators, that is, $0 \leqslant A$ with $\mathrm{tr}(A) = 1$.

Mathematics subject classification (2010): 33B99, 47A05.

Keywords and phrases: Concave function, density operator, inequality.

REFERENCES

[1] T. ANDO, *Majorization, doubly stochastic matrices and comparison of eigenvalues*, Linear Algebra Appl., vol. 118 (2) (1989), pp. 163–248.

[2] R. BHATIA, *Matrix Analysis*, Springer-Verlag, New York (1997).

[3] E. CARLEN, *Trace Inequalities and Quantum Entropy: An Introductory Course Contemporary Math.*, vol. 529 (2) (2010), pp. 73–140.

[4] L. K. HUA, *Inequalities involving determinants*, Acta Math. Sinica, vol. 5 (2) (1955), pp. 463–470.

[5] Z. H. MA, W. G. YUAN, M. L. BAO, X. D. ZHANG, *A new entanglement measure: D-concurrence*, Quantum Inf. Comput., vol. 11 (2) (2011), pp. 70–78.

Paper MIA-18-43

[6] M. A. NIELSEN, *An introduction of majorization and its applications to quantum mechanics*, Queensland, Australia, (2002).
[7] R. T. ROCKAFELLAR, *Convex Analysis*, Princeton University Press, Princeton, (1972).