

## ON THE $q$ -BETA FUNCTION INEQUALITIES

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*Abstract.* In this paper, we provide some inequalities for the  $q$ -beta function with  $q > 0$  by means of calculus and  $q$ -Čebyšev-Grüss type integral inequalities. When letting  $q \rightarrow 1$ , some of our results approach to results obtained by some authors and the others are shown to be new. *Mathematics subject classification (2010):* 33D05, 26D07, 26A48.

*Keywords and phrases:* Inequalities,  $q$ -beta function,  $q$ -gamma function, monotonic function,  $q$ -Čebyšev-Grüss integral inequalities.

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