SOME GEOMETRIC PROPERTIES OF MUSIELAK–ORLICZ SEQUENCE SPACES GENERATED BY DE LA VALLÉE–POUSSIN MEANS

ATANU MANNA AND P. D. SRIVASTAVA

Abstract. Necessary and sufficient conditions for the non-triviality of a Musielak-Orlicz sequence space $V_\Phi(\lambda)$ generated by the de la Vallée-Poussin means are obtained. Topological properties such as completeness, separability, order continuity are characterized for the space $V_\Phi(\lambda)$. Finally, criteria for the coordinatewise uniformly Kadec-Klee property and the Uniform Opial property are obtained.

REFERENCES

[1] A. KAMIŃSKA, *Uniform rotundity of Musielak-Orlicz sequence spaces*, Journal of Approximation Theory, **47**, 4 (1986), 302–322.

[2] H. HUDZIK, Y. N. YE, *Support functionals and smoothness in Musielak-Orlicz sequence spaces endowed with Luxemberg norm*, Comment. Math. Univ. Carolinae, **31**, 4 (1990), 661–684.

[3] J. MUSIELAK, *Orlicz spaces and Modular spaces*, Springer Lecture notes in Math., **1034**, Springer, 1983.

[4] J. P. GOSSEZ, E. LAMI DOZO, *Some geometric properties related to fixed point theory for non expansive mappings*, Pacific Journal of Math., **40**, (1972), 565–573.

[5] L. LEINDLER, *Über die verallgemeinerte de la Vallée-Poussinsche Summierbarkeit allgemeiner Orthogonalreihen*, Acta Math. Acad. Sci. Hungar., **16**, (1965), 375–387.

[6] L. V. KANTOROVICH, G. P. AKILOV, *Functional Analysis*, Pergamon Press and "Nauka", Second Edition, 1982.

[7] M. I. KADETS, *The relation between some properties of convexity of the unit ball of a Banach space*, Functional Analysis and Its Application, **16**, 3 (1982), 204–206 (translation).

[8] M. MURSALEEN, *λ-statistical convergence*, Math. Slovaca, **50**, 1 (2000), 111–115.

[9] N. PETROT, S. SUANTAI, *Some geometric properties in Cesàro-Orlicz Spaces*, Science Asia, **31**, (2005), 173–177.

[10] N. PETROT, S. SUANTAI, *Uniform Opial properties in generalized Cesàro sequence spaces*, Nonlinear Analysis, **63**, (2005), 1116–1125.

[11] N. ŞIMŞEK, E. SAVAŞ, V. KARAKAYA, *Some geometric and topological properties of a new sequence spaces defined by De La Vallée-Poussin Mean*, Journal of Computational Analysis and Applications, **12**, 4 (2010), 768–779.

[12] N. ŞIMŞEK, *On some geometric of sequence space defined by De La Vallée-Poussin Mean*, Journal of Computational Analysis and Applications, **13**, 3 (2011), 565–573.

[13] P. FORALEWSKI, H. HUDZIK AND A. SZYMASZKIEWICZ, *Some remarks on Cesàro-Orlicz spaces*, Mathematical Inequalities and Applications, **13**, 2 (2010), 363–386.

[14] P. FORALEWSKI, H. HUDZIK, *On some geometrical and topological properties of generalized Calderón-Lozanovskiǐ sequence spaces*, Houston Journal of Mathematics, **25**, 3 (1999), 523–542.

[15] P. FORALEWSKI, H. HUDZIK, A. SZYMASZKIEWICZ, *Local rotundity structure of Cesàro-Orlicz sequence spaces*, J. Math. Anal. Appl., **345**, 1 (2008), 410–419.

[16] R. HUFF, *Banach spaces which are nearly uniformly convex*, Rockey Mountain J. Math., **10**, (1980), 743–749.

[17] R. WANGKEEREE, *On property* (k-*NUC*) *in Cesàro-Musielak-Orlicz sequence spaces*, Thai Journal of Mathematics, **1**, (2003), 119–130.

[18] S. T. CHEN, *Geometry of Orlicz spaces*, Dissertationes Math., **356**, (1996), 1–204.

[19] S. PRUS (EDS. W. A. KIRK AND B. SIMS), *Geometrical background of metric fixed point theory*, Handbook of Metric Fixed Point Theory, Kluwer Academic Publishers, Dordrecht, 2001, 93–132.

[20] S. PRUS, *Banach spaces with Uniform Opial property*, Nonlinear Anal. Theory Appl., **18**, 8 (1992), 697–704.

[21] T. ZHANG, *The coordinatewise Uniformly Kadec-Klee property in some Banach spaces*, Siberian Mathematical Journal, **44**, 2 (2003), 363–365.

[22] W. A. J. LUXEMBURG, *Banach function spaces*, Thesis, Delft, (1955).

[23] Y. CUI, H. HUDZIK, N. PETROT, S. SUANTAI AND A. SZYMASZKIEWICZ, *Basic topological and geometric properties of Cesàro-Orlicz spaces*, Proc. Indian Acad. Sci. (Math. Sci.), **115**, 4 (2005), 461–476.

[24] Y. CUI, H. HUDZIK, *Maluta's coefficient and Opial's properties in Musielak-Orlicz sequence spaces equipped with the Luxemberg norm*, Nonlinear Analysis, **35**, (1999), 475–485.

[25] Y. CUI, H. HUDZIK, *On the Uniform Opial property in some modular sequence spaces*, Func. Approx. Comment., **26**, (1998), 93–102.

[26] Z. OPIAL, *Weak convergence of the sequence of successive approximations for non expansive mappings*, Bull. Amer. Math. Soc., **73**, (1967), 591–597.

Mathematical Inequalities & Applications
www.ele-math.com
mia@ele-math.com