

INTERPOLATION WITH A PARAMETER FUNCTION AND INTEGRABLE FUNCTION SPACES WITH RESPECT TO VECTOR MEASURES

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Abstract. We establish interpolation formulae for different compatible couples formed by spaces of scalar integrable functions with respect to a vector measure in connection with a parameter function.

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