

CENTERED CONVEX BODIES AND INEQUALITIES FOR CROSS-SECTION MEASURES

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Abstract. The purpose of this paper is to establish some new results on cross-section measures of centered convex bodies. More precisely, we show some connections between inequalities referring to cross-section measures and well-known affine isoperimetric inequalities. Based on this, we derive affine inequalities involving also new characterizations of ellipsoids. In addition, related results on three-dimensional zonoids are obtained. Some of our results are also interesting from the viewpoint of the geometry of finite dimensional real Banach spaces.

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