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VECTOR MAJORIZATION AND SCHUR–CONCAVITY OF SOME SUMS GENERATED BY THE JENSEN AND JENSEN–MERCER FUNCTIONALS

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Abstract. In this paper we study a vector majorization ordering for comparing two m-tuples of vectors of a real linear space. This extends the classical approach of (scalar) majorization theory for comparing m-tuples of scalars in $\mathbf{R}$. We prove a Sherman type inequality for a vector-valued $\leqslant_C$-convex function f, where $\leqslant_C$ is a cone ordering. In consequence, we obtain a Hardy-Littlewood-Pólya-Karamata type inequality generated by m-tuples of vectors in a vector space. As applications, we present majorization generalizations of the superadditivity properties of the Jensen and Jensen-Mercer functionals generated by a convex function f. In addition, we show that some sums generated by the Jensen and Jensen-Mercer functionals are Schur-concave with respect to their weight vectors. We also give interpretations of the obtained results for tridiagonal doubly stochastic matrices and doubly stochastic circular matrices.

Mathematics subject classification (2010): 52A40, 06F20, 26B25, 15A39.

Keywords and phrases: Convex function, Jensen functional, Jensen-Mercer functional, sub-/super-additive function, vector majorization, cone ordering, Schur-convex/concave function, column stochastic matrix, doubly stochastic matrix, doubly stochastic circular matrix, tridiagonal doubly stochastic matrix.

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