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ENDPOINT ESTIMATES FOR COMMUTATORS OF INTRINSIC SQUARE FUNCTIONS IN MORREY TYPE SPACES

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Abstract. In this paper, the boundedness properties of commutators generated by b and intrinsic square functions in the endpoint case are discussed, where $b \in BMO(\mathbb{R}^n)$. We first establish the weighted weak $L\log L$-type estimates for these commutator operators. Furthermore, we will prove endpoint estimates of commutators generated by $BMO(\mathbb{R}^n)$ functions and intrinsic square functions in Morrey type spaces. In particular, we can obtain endpoint estimates of these commutators in the weighted Morrey spaces $L^{1,\kappa}(w)$ for $0<\kappa<1$ and $w \in A_1$, and in the generalized Morrey spaces $L^{1,\Theta}$, where Θ is a growth function on $(0,+\infty)$ satisfying the doubling condition.

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REFERENCES

[1] D. R. ADAMS, *A note on Riesz potentials*, Duke Math. J, **42** (1975), 765–778.
[2] J. ALVAREZ, R. J. BAGBY, D. S. KURTZ AND C. PÉREZ, *Weighted estimates for commutators of linear operators*, Studia Math, **104** (1993), 195–209.
[3] F. CHIARENZA AND M. FRASCA, *Morrey spaces and Hardy–Littlewood maximal function*, Rend. Math. Appl, **7** (1987), 273–279.
[4] Y. DING, S. Z. LU AND K. YABUTA, *On commutators of Marcinkiewicz integrals with rough kernel*, J. Math. Anal. Appl, **275** (2002), 60–68.
[5] J. DUOANDIKOETXEA, *Fourier Analysis*, American Mathematical Society, Providence, Rhode Island, 2000.
[6] J. GARCIA-CUERVA AND J. L. RUBIO DE FRANCIA, *Weighted Norm Inequalities and Related Topics*, North-Holland, Amsterdam, 1985.
[7] V. S. GULIYEV, *Boundedness of the maximal, potential and singular operators in the generalized Morrey spaces*, J. Inequal. Appl., Article ID 503948, (2009).
[8] V. S. GULIYEV, S. S. ALIYEV AND T. KARAMAN, *Boundedness of a class of sublinear operators and their commutators on generalized Morrey spaces*, Abstr. Appl. Anal., Article ID 356041, (2011).
[9] V. S. GULIYEV, S. S. ALIYEV, T. KARAMAN AND P. S. SHUKUROV, *Boundedness of sublinear operators and commutators on generalized Morrey spaces*, Integr. Equ. Oper. Theory, **71** (2011), 327–355.
[10] F. JOHN AND L. NIRENBERG, *On functions of bounded mean oscillation*, Comm. Pure Appl. Math, **14**(1961), 415–426.
[11] Y. KOMORI AND S. SHIRAI, *Weighted Morrey spaces and a singular integral operator*, Math. Nachr, **282** (2009), 219–231.
[12] S. Z. LU, D. C. YANG AND Z. S. ZHOU, *Sublinear operators with rough kernel on generalized Morrey spaces*, Hokkaido Math. J, **27** (1998), 219–232.
[13] T. MIZUHARA, *Boundedness of some classical operators on generalized Morrey spaces*, Harmonic Analysis, ICM-90 Satellite Conference Proceedings, Springer-Verlag, Tokyo, (1991), 183–189.
[14] C. B. MORREY, *On the solutions of quasi-linear elliptic partial differential equations*, Trans. Amer. Math. Soc, **43** (1938), 126–166.

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[15] E. NAKAI, *Hardy–Littlewood maximal operator, singular integral operators and Riesz potentials on generalized Morrey spaces*, Math. Nachr., **166** (1994), 95–103.
[16] B. MUCKENHOUPT, *Weighted norm inequalities for the Hardy maximal function*, Trans. Amer. Math. Soc, **165** (1972), 207–226.
[17] J. PEETRE, *On the theory of $\mathcal{L}_{p,\lambda}$ spaces*, J. Funct. Anal, **4** (1969), 71–87.
[18] C. PÉREZ, *Endpoint estimates for commutators of singular integral operators*, J. Funct. Anal, **128** (1995), 163–185.
[19] C. PÉREZ AND G. PRADOLINI, *Sharp weighted endpoint estimates for commutators of singular integrals*, Michigan Math. J., **49** (2001), 23–37.
[20] C. PÉREZ AND R. TRUJILLO-GONZÁLEZ, *Sharp weighted estimates for multilinear commutators*, J. London Math. Soc., **65** (2002), 672–692.
[21] M. M. RAO AND Z. D. REN, *Theory of Orlicz Spaces*, Marcel Dekker, New York, 1991.
[22] E. M. STEIN, *Singular Integrals and Differentiability Properties of Functions*, Princeton Univ. Press, Princeton, New Jersey, 1970.
[23] H. WANG, *Intrinsic square functions on the weighted Morrey spaces*, J. Math. Anal. Appl, **396** (2012), 302–314.
[24] H. WANG, *Weak type estimates for intrinsic square functions on weighted Morrey spaces*, Anal. Theory Appl, **29** (2013), 104–119.
[25] H. WANG, *Boundedness of intrinsic square functions on generalized Morrey spaces*, Georgian Math. J, **21** (2014), 351–367.
[26] H. WANG, *Boundedness of vector-valued intrinsic square functions in Morrey type spaces*, J. Funct. Spaces., Article ID 923680, (2014), 8 pages.
[27] H. WANG, *The boundedness of intrinsic square functions on the weighted Herz spaces*, J. Funct. Spaces., Article ID 274521, (2014), 14 pages.
[28] M. WILSON, *The intrinsic square function*, Rev. Mat. Iberoamericana, **23** (2007), 771–791.
[29] M. WILSON, *Weighted Littlewood–Paley Theory and Exponential-Square Integrability*, Lecture Notes in Math, Vol 1924, Springer-Verlag, 2007.
[30] P. ZHANG, *Weighted endpoint estimates for commutators of Marcinkiewicz integrals*, Acta Math. Sinica (Engl. Ser), **26** (2010), 1709–1722.