A UNIFIED TREATMENT OF HILBERT–TYPE INEQUALITIES INVOLVING THE HARDY OPERATOR

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Abstract. In this paper we shall provide a unified treatment of Hilbert-type inequalities involving the Hardy operator with a general homogeneous kernel. Furthermore, we establish some new Hilbert-type inequalities involving the integral operator. We shall show that the constants in our results are best possible.

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