Mathematical
Inequalities
& Applications
Volume 18, Number 3 (2015), 845–858 doi:10.7153/mia-18-61

ON THE ORDER OF MAGNITUDE OF FOURIER TRANSFORM

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Abstract. For a Lebesgue integrable complex-valued function f defined on $\mathbb{R}$, let $\hat{f}$ be its Fourier transform. The Riemann-Lebesgue lemma says that $\hat{f}(t) \to 0$ as $|t| \to \infty$. But in general, there is no definite rate at which the Fourier transform tends to zero. In fact, the Fourier transform of an integrable function can tend to zero as slowly as we wish. Therefore, it is interesting to know for functions of which subclasses of $L^1(\mathbb{R})$ there is a definite rate at which the Fourier transform tends to zero. In this paper, we determine this rate for functions of bounded variation on $\mathbb{R}$. We also determine such rate of Fourier transform for functions of bounded variation in the sense of Vitali defined on $\mathbb{R}^N$ $(N \in \mathbb{N})$.

Mathematics subject classification (2010): 42A38, 42B10, 26A12, 26A45, 26B30, 26D15.

Keywords and phrases: Fourier transform, function of bounded variation over $\mathbb{R}$, function of bounded variation over $\mathbb{R}^2$, function of bounded variation over $\mathbb{R}^N$, order of magnitude.

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