

## A GENERALIZED CAUCHY–SCHWARZ INEQUALITY

MOWAFFAQ HAJJA

*Abstract.* In the course of realizing certain triangle centers as points that minimize certain quantities, C. Kimberling and P. Moses, in *Math. Mag.* **85** (2012) 221–227, discovered an inequality in three variables that generalizes the Cauchy-Schwarz inequality, and made a conjecture regarding a generalization of that inequality to an arbitrary number of variables. In this paper, we give a proof of a stronger form of that conjecture.

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