Mathematical
Inequalities
& Applications
Volume 18, Number 3 (2015), 941–947 doi:10.7153/mia-18-70

ON A DISCRETE WEIGHTED MIXED ARITHMETIC–GEOMETRIC MEAN INEQUALITY

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Abstract. Let $n \geqslant 2$. For $1 \leqslant i \leqslant n$, let x_i, $w_i \geqslant 0$ with $w_1 > 0$. Further let $W_i = \sum_{k=1}^{i} w_k$, $M_{i,1} = \sum_{k=1}^{i} w_k x_k / W_k$, $M_{i,0} = \prod_{k=1}^{i} x_k^{w_k/W_k}$, $M_{i,1}(\mathbf{M}_{i,0}) = \sum_{k=1}^{i} w_k M_{k,0}/W_k$, $M_{i,0}(\mathbf{M}_{i,1}) = \prod_{k=1}^{i} M_{k,1}^{w_k/W_k}$. A result of Holland states that when $W_{n-1}^2 \geqslant w_n \sum_{i=1}^{n-2} W_i$, then

$$W_{n-1}\Big(M_{n-1,0}(\mathbf{M}_{n-1,1}) - M_{n-1,1}(\mathbf{M}_{n-1,0})\Big) \leqslant W_n\Big(M_{n,0}(\mathbf{M}_{n,1}) - M_{n,1}(\mathbf{M}_{n,0})\Big).$$

The above result implies a discrete weighted mixed arithmetic-geometric mean inequality. In this paper, we extend the validity of the above inequality by considering the case $W_{n-1}^2 < w_n \sum_{i=1}^{n-2} W_i$.

Mathematics subject classification (2010): Primary 26D15.
Keywords and phrases: Arithmetic-geometric mean, mixed-mean inequality.

REFERENCES

[1] P. S. BULLEN, *Inequalities due to T. S. Nanjundiah*, *Recent progress in inequalities*, Kluwer Acad. Publ., Dordrecht, 1998, 203–211.
[2] P. GAO, *A note on mixed-mean inequalities*, J. Inequal. Appl., **2010** (2010), Art. ID 509323, 8 pp.
[3] F. HOLLAND, *An inequality between compositions of weighted arithmetic and geometric means*, JIPAM. J. Inequal. Pure Appl. Math., **7** (2006), Article 159, 8 pp. (electronic).
[4] K. KEDLAYA, *A weighted mixed-mean inequality*, Amer. Math. Monthly, **106** (1999), 355–358.
[5] T. S. NANJUNDIAH, *Sharpening of some classical inequalities*, Math Student, **20** (1952), 24–25.
[6] C. TARNAVAS AND D. TARNAVAS, *An inequality for mixed power means*, Math. Inequal. Appl., **2** (1999), 175–181.

Paper MIA-18-70