

A GENERALIZED ČEBYŠEV FUNCTIONAL FOR THE RIEMANN–STIELTJES INTEGRAL

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Abstract. Sharp bounds for a generalised Čebyšev functional for the Riemann–Stieltjes integral are given.

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