WEIGHTED INEQUALITIES RELATED TO A MUCKENHOUPT AND WHEEDEN PROBLEM FOR ONE–SIDE SINGULAR INTEGRALS

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Abstract. In this paper we obtain for T^+, a one-sided singular integral given by a Calderón-Zygmund kernel with support in $(-\infty, 0)$, a $L^p(w)$ bound when $w \in A_1^+$. In [A. K. Lerner, S. Ombrosi and C. Pérez, A_1 *Bounds for Calderón-Zygmund operators related to a problem of Muckenhoupt and Wheeden*, Math. Res. Lett. **16** (2009), no. 1, 149–156.], the authors proved that this bound is sharp with respect to $||w||_{A_1}$ and with respect to p. We also give a $L^{1,\infty}(w)$ estimate, for a related problem of Muckenhoupt and Wheeden for $w \in A_1^+$. We improve the classical results, for one-sided singular integrals, by putting in the inequalities a wider class of weights.

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