

ON BACKWARD ALUTHGE ITERATES OF HYPONORMAL OPERATORS

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Abstract. In this paper we study several remarkable properties of the backward Aluthge iterates of a hyponormal operator. In particular, we show that, under suitable conditions, operators in $BAIH(k)$ admit a moment sequence and have nontrivial hyperinvariant subspaces.

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