

LOGARITHMIC COMPLEMENTARY MEANS AND AN EXTENSION OF CARLSON'S LOG

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Abstract. The invariance equality $L \circ (\mathcal{M}, \mathcal{N}) = L$, where L is the logarithmic mean, and where the unsymmetric compound means $\mathcal{M} = A \circ (P_1, G)$, $\mathcal{N} = A \circ (P_2, G)$ are built with the arithmetic A , geometric G , and projective means P_1, P_2 , is called “Carlson’s log” and is important in iteration of means. In the present paper we present effective and simple 1-parameter families of unsymmetric means $M_t, N_t : (0, \infty)^2 \rightarrow (0, \infty)$ such that, for all $t \in (-1, 1)$,

$$L \circ (M_t, N_t) = L$$

and

$$\mathcal{M} = M_{\frac{1}{2}}, \quad \mathcal{N} = N_{\frac{1}{2}}.$$

Existence of elementary (simple) symmetric means M and N such that $L \circ (M, N) = L$ and $M \neq L$ is posed as an open problem.

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