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ASYMPTOTICS OF AN EULER–LAGRANGE EQUATION ASSOCIATED WITH EXTREMAL FUNCTIONS OF THE HARDY–SOBOLEV INEQUALITY

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Abstract. In this paper, we are concerned with the Hardy type integral equation

$$u(x)=\int_{R^n}\frac{u^p(y)dy}{|y|^l|x-y|^{n-\alpha}},\quad u>0\quad\text{in}\quad R^n,$$

where $n\geqslant 1$, $\alpha\in(0,n)$, $p>1$ and $l\geqslant\alpha$. Such an equation is related to the extremal functions of the Hardy-Sobolev inequality. If $l>\alpha$, then there is not any bounded positive solution for such an integral equation. If $l=\alpha$, the fast and the slow decay rates are discussed according to whether the nonnegative solutions possess some integrability.

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