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PARTIAL SUMS OF THE NORMALIZED LOMMEL FUNCTIONS

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Abstract. The aim of the present paper determine the ratio of the normalized Lommel functions $\mathscr{L}_{\mu,\nu}$ of the form (**??**) to its sequence of partial sums $\left(\mathscr{L}_{\mu,\nu}\right)_m(z) = z + \sum_{n=1}^{m} a_n z^{n+1}$ when the coefficients of $\mathscr{L}_{\mu,\nu}$ satisfy some conditions. Furthermore we investigate the radii of univalency, starlikeness, convexity and close-to-convexity of the partial sums $\left(\mathscr{L}_{\mu,\nu}\right)_m(z)$. Computational and graphical usages of Maple (Version 17) as well as geometrical descriptions of the image domains in several illustrative examples are also presented.

Mathematics subject classification (2010): 30C45, 33C10.

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