

ON A MULTIDIMENSIONAL HILBERT-TYPE INTEGRAL INEQUALITY WITH LOGARITHM FUNCTION

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Abstract. By the use of the transfer formula, the methods of weight functions and technique of Real Analysis, a multidimensional Hilbert-type integral inequality with a few parameters and a best possible constant factor related to the kernel of logarithm function is given. The equivalent forms and some reverses are obtained. The operator expressions and a few particular results related to the kernels of non-homogeneous and homogeneous are considered.

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