GENERALIZED ROLEWICZ THEOREM
FOR CONVEXITY OF HIGHER ORDER

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Abstract. In this paper we prove that if a real function f satisfies the inequality $[x_0, x_1, \ldots, x_{n+1}; f]$ $+ \varphi(x_{n+1} - x_0) \geqslant 0$ for all $x_0 < x_1 < \ldots < x_n < x_{n+1}$ with some fixed positive integer n and non-negative function φ fulfilling $\lim_{h \to 0+} \varphi(h) = 0$, then f is convex of order n, i.e., f satisfies the former inequality with $\varphi = 0$ as well.

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