

ON BOUNDEDNESS OF FRACTIONAL MAXIMAL OPERATOR IN WEIGHTED $L^{p(\cdot)}$ SPACES

FARMAN I. MAMEDOV AND YUSUF ZEREN

Abstract. In this paper, we derive some sufficient conditions for the boundedness of the fractional maximal operator in the weighted variable exponent Lebesgue spaces $L^{p(\cdot)}$, where Sawyer's type pair of modular conditions are proposed on a weight functions and it is assumed a local log-regularity and a decay condition on the exponent function $p(\cdot)$.

Mathematics subject classification (2010): 42A05, 42B25, 26D10, 35A23.

Keywords and phrases: Fractional maximal operator, weighted Lebesgue space, variable exponent.

REFERENCES

- [1] M. ASIF, V. KOKILASHVILI, AND A. MESKHI, *Boundedness criteria for maximal functions and potentials on the half-space in weighted Lebesgue spaces with variable exponent*, Int. Trans. Spec. Funct. **20** (2009), 805–819.
- [2] C. CAPONE, D. CRUZ-URIBE SFO AND A. FIORENZA, *The fractional maximal operator on variable L^p spaces*, Revista Math. Iberoamericana. **23**, 3 (2007), 747–770.
- [3] D. CRUZ-URIBE, L. DIENING, A. FIORENZA, *A new proof of the boundedness of the maximal operator on variable Lebesgue spaces*, Bol. Unione Mat. Ital. (9) **2**, 1 (2009), 151–173.
- [4] D. CRUZ-URIBE, A. FIORENZA AND C. J. NEUGEBAUER, *The maximal function on variable L^p spaces*, Ann. Acad. Scient. Fenn. Mathematics. **28** (2003), 223–238.
- [5] D. CRUZ-URIBE, *New proofs of two-weight norm inequalities for a maximal operator*, Georgian Math. Journal. **7**, 1 (2000), 33–42.
- [6] D. CRUZ-URIBE, A. FIORENZA AND C. J. NEUGEBAUER, *Weighted norm inequalities for the maximal operator on variable Lebesgue spaces*, J. Math. Anal. Appl. **394** (2012) 744–760.
- [7] D. CRUZ-URIBE, A. FIORENZA, *Variable Lebesgue Spaces. Foundations and Harmonic Analysis*, Series: Applied and Numerical Harmonic Analysis, Birkhäuser, Basel, 2013, IX, 312 p.
- [8] L. DIENING AND P. HASTO, *Muckenhoupt weights in variable exponent spaces*, Preprint, 2010.
- [9] L. DIENING, *Maximal functions on Musielak-Orlicz spaces and generalized Lebesgue spaces*, Bulletin des Sciences Mathématiques **129**, 8 (2005), 657–700.
- [10] L. DIENING, *Maximal function on generalized Lebesgue spaces $L^{p(\cdot)}$* , Math. Inequal. Appl. **7**, 2 (2004), 245–253.
- [11] D. E. EDMUNDS, V. M. KOKILASHVILI AND A. MESKHI, *Two-weight estimates in $L^{p(x)}$ spaces with applications to Fourier series*, Houston J. Math. **35**, 2 (2009), 665–689.
- [12] J. GARCIA-CUERVA AND J. M. MARTELL, *Two-weight norm inequalities for maximal operators and fractional integrals on non-homogeneous spaces*, Indiana Univ. Math. J. **50**, 3 (2001), 1241–1280.
- [13] A. HARMAN AND F. I. MAMEDOV, *On boundedness of weighted Hardy operator in $L^{p(\cdot)}$ and regularity condition*, J. Ineq. Appl. 2010, 2010:837951 (6 December 2010)
- [14] P. HARJULEHTO, P. HÄSTÖ AND M. PERE, *Variable exponent Lebesgue spaces on metric spaces: the Hardy-Littlewood maximal operator*, Real Anal. Exchange. **30** (2004–2005), 87–104.
- [15] V. M. KOKILASHVILI AND A. MESKHI, *Weighted criteria for generalized fractional maximal functions and potentials in Lebesgue spaces with variable exponent*, Integral Transforms and Special Functions, **18**, 9 (2007), 609–628
- [16] V. M. KOKILASHVILI AND A. MESKHI, *Two-weight inequalities for fractional maximal functions and singular integrals in $L^{p(\cdot)}$ spaces*, J. Math. Sci. **173**, 6 (2011), 656–673.

- [17] V. M. KOKILASHVILI AND S. G. SAMKO, *Operators of harmonic analysis in weighted spaces with non-standard growth*, J. Math. Anal. Appl. **352**, 1 (2009), 15–34 (2008).
- [18] V. KOKILASHVILI AND S. SAMKO, *Boundedness of maximal operators and potential operators on Carleson curves in Lebesgue spaces with variable exponent*, Acta Mathematica Sinica, **24**, 11 (2008), 1775–1800.
- [19] V. KOKILASHVILI AND S. SAMKO, *The maximal operator in weighted variable exponent spaces on metric spaces*, Georgian Math. J. **15** (2008), p. 683–712.
- [20] V. M. KOKILASHVILI AND S. G. SAMKO, *The maximal operator in weighted variable spaces on metric spaces*, Proc. A. Razmadze Math. Inst., **144** (2007), 134–137.
- [21] V. M. KOKILASHVILI, S. G. SAMKO AND N. SAMKO, *The maximal operator in weighted variable spaces $L^{p(\cdot)}$* , Journal of Function Spaces and Applications, **5** (2007), 3, 299–317.
- [22] V. M. KOKILASHVILI AND S. G. SAMKO, *Maximal and fractional operators in weighted $L^{p(x)}$ -spaces*, Revista Mathematica Iberoamericana, **20**, 2 (2004), 493–515.
- [23] O. KOVACIK AND J. RAKOSNIK, *On spaces $L^{p(x)}$ and $W^{L^{p(x)}}$* , Czechoslovak Math. J. **41** (116) (1991), 592–618.
- [24] A. K. LERNER, *Some remarks on the Hardy–Littlewood maximal function on variable L^p spaces*, Math. Z. **241**, 3 (2005), 509–521.
- [25] F. I. MAMEDOV AND Y. ZEREN, *On a two weighted estimation of maximal operator in the Lebesgue space with variable exponent*, Anali di Matematica, **190**, 2 (2010), 263–275.
- [26] F. I. MAMEDOV AND Y. ZEREN, *Two-weight inequalities for the maximal operator in a Lebesgue space with variable exponent*, J. Math. Sci. **173**, 6 (2011), 701–716.
- [27] A. NEKVINDA, *Hardy–Littlewood maximal operator on $L^{p(\cdot)}(\mathbb{R}^n)$* , Math. Inequal. Appl., **7**, 2 (2004), 255–265.
- [28] L. PICK AND M. RUZICKA, *An example of a space $L^{p(x)}$ on which the Hardy–Littlewood maximal operator is not bounded*, Expo. Math. **19**, 4 (2001), 369–371.
- [29] Y. RAKOTONDRAZIMBA, *On Muckenhoupt and Sawyer conditions for maximal operators*, Publ. Mat. **37** (1993), 57–73.
- [30] E. T. SAWYER, *A characterization of a two-weight norm inequality for maximal operators*, Studia Math. **75** (1982), 1–11.
- [31] S. SAMKO, *Convolution type operators in $L^{p(x)}$* , Integr. Transf. and Spec. Funct, **7**, 1–2 (1998), 123–144.