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FRACTIONAL MAXIMAL OPERATOR AND FRACTIONAL INTEGRAL OPERATOR ON ORLICZ–LORENTZ SPACES

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Abstract. In this paper, we prove the characterization of the weighted modular inequalities for the fractional maximal operator M_α $(0 \leqslant \alpha < n)$ on the Orlicz-Lorentz spaces by atomic decomposition which induces a sufficient condition of the boundedness for this operator on the Orlicz-Lorentz spaces. And we also find the characterization of the weighted modular inequalities for the fractional integral operator I_α $(0 < \alpha < n)$ on the Orlicz-Lorentz spaces in certain case which leads to a sufficient condition of the boundedness for I_α $(0 < \alpha < n)$.

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REFERENCES

[1] C. BENNETT AND R. SHARPLEY, *Interpolation of operators*, Pure and Applied Mathematics, Vol. **129**, Academic Press, Inc., Boston, MA, 1988.
[2] C. BENNETT AND K. RUDNICK, *On Lorentz-Zygmund spaces*, Dissertationes Mathematicae, CLXXV, Warszawa, 1980.
[3] S. BLOOM AND R. KERMAN, *Weighted L_Φ modular inequalities for operators of Hardy type*, Studia Math. **110**, 1 (1994), 35–52.
[4] M. J. CARRO, J. A. RAPOSO AND J. SORIA, *Recent developements in the theory of Lorentz spaces and weighted inequalities*, Mem. Amer. Math. Soc. **187**, 877 (2007).
[5] M. J. CARRO, J. SORIA, *Weighted Lorentz spaces and the Hardy operator*, J. Funct. Anal. **112**, 112 (1993), 480–494.
[6] A. CIANCHI AND D. E. EDMUNDS, *On fractional integration in weighted Lorentz spaces*, Quarterly J. Math. Oxford Ser. **48**, 3 (1997), 439–451.
[7] A. KAMIŃSKA AND M. MASTYŁO, *Abstract duality Sawyer formula and its applications*, Monatsh. Math. **151**, 3 (2007), 223–245.
[8] R. KERMAN AND A. TORCHINSKY, *Integral inequalities with weights for the Hardy maximal function*, Studia Math. **71**, 3 (1982), 277–284.
[9] V. KOKILASHVILI, *Weighted inequalities for maximal functions and fractional integrals in Lorentz spaces*, Math. Nachr. **133**, 1987, 33–42.
[10] M. A. KRASNOSEL'SKII AND Y. B. RUTICKII, *Convex functions and Orlicz spaces (translation)*, P. Noordhoff Ltd., Groningen, 1961.
[11] M. KRBEC AND L. PICK, *On imbeddings between weighted Orlicz spaces*, Z. Anal. Anwendungen **10**, 1 (1991), 107–117.
[12] M. KŘEPELA, *Convolution inequalities in weighted Lorentz spaces*, Math. Inequal. Appl. **17**, 4 (2014), 1201–1223.
[13] H. P. HEINIG AND Q. LAI, *Weighted modular inequalities for Hardy-type operators on monotone functions*, J. Inequal. Pure Appl. Math. **1**, 1 (2000), Article 10, 25 pp. (electronic).
[14] H. L. LI, *Hardy-type inequalities on strong and weak Orlicz-Lorentz spaces*, Sci. China Math. **55**, 12 (2012), 2493–2505.
[15] W. A. J. LUXEMBURG, *Banach Function spaces*, Thesis, Delft Technical Univ., 1955.
[16] G. G. LORENTZ, *Some new functional spaces*, Ann. of Math. **51**, 1950, 37–55.

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[17] G. G. LORENTZ, *On the theory of spaces* Λ, Pacific J. Math. **1**, 1951, 411–429.
[18] L. MALIGRANDA, *Indices and interpolation*, Dissert. Math. **234**, 1984, 1–49.
[19] M. MASTYŁO, *Interpolation of linear operators in Calderón-Lozanovskiĭ spaces*, Comment. Math. Prace Mat. **26**, 2 (1986), 247–256.
[20] R. O'NEIL, *Convolution operators and* $L(p,q)$ *spaces*, Duke Math. J. **30**, 1963, 129–142.
[21] R. O'NEIL, *Fractional integration in Orlicz spaces*, I, Trans. Amer. Math. Soc. **115**, 1965, 300–328.
[22] E. NURSULTANOV AND S. TIKHONOV, *Convolution Inequalities in Lorentz Spaces*, J. Fourier Anal. Appl. **17**, 2011, 486–505.
[23] W. ORLICZ, *Über eine gewisse Klasse von Räumen vom Typus B*, Bull, Intern. Acad. Pol. **8**, 1932, 207–220.
[24] Y. RAKOTONDRATSIMBA, *Weighted Inequalities for the Fractional Maximal Operator in Lorentz Spaces via Atomic Decomposition of Tent Spaces*, Z. Anal. Anwend. **16**, 2 (1997), 263–280.
[25] M. M. RAO AND Z. D. REN, *Theory of Orlicz Spaces, Marcel Dekker*, Inc., New York, Basel, 1991.
[26] E. SAWYER, *A characterization of a two-weight norm inequality for maximal operators*, Studia Math. **75**, 1982, 1–11.
[27] E. SAWYER, *Boundedness of classical operators on classical Lorentz spaces*, Studia Math. **96**, 1990, 145–158.
[28] R. SHARPLEY, *Fractional Integration in Orlicz Spaces*, Proc. Amer. Math. Soc. **59**, 1 (1976), 99–106.
[29] J. SORIA, *Weighted tent spaces*, Math. Nach. **155**, 1992, 231–256.
[30] V. D. STEPANOV, *Weighted norm inequalities for integral operators and related topics*, Nonlinear Anal., Funct. Space Appl. **5**, 1994, 139–175.