

## NEW MULTIPLICATIVE HIGHER ORDER DYNAMIC INEQUALITIES OF OPIAL TYPE

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*Abstract.* In this paper, we prove some new multiplicative dynamic inequalities of Opial type on a time scale  $\mathbb{T}$ . The main results will be proved by using Hölder's inequality, the chain rule and some basic dynamic inequalities designed and proved for this purpose. As special cases, we will derive some continuous and discrete inequalities from the main results.

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