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ON THE UPPER AND LOWER ESTIMATES OF NORMS IN VARIABLE EXPONENT SPACES

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Abstract. In the present paper we investigate some geometrical properties of the norms in Banach function spaces. Particularly there is shown that if exponent $1/p(\cdot)$ belongs to $BLO^{1/\log}$ then for the norm of corresponding variable exponent Lebesgue space we have the following lower estimate

$$\left\|\sum \chi_Q \|f\chi_Q\|_{p(\cdot)}/\|\chi_Q\|_{p(\cdot)}\right\|_{p(\cdot)} \leqslant C\|f\|_{p(\cdot)}$$

where $\{Q\}$ defines disjoint partition of $[0;1]$. Also we have constructed variable exponent Lebesgue space with above property which does not possess following upper estimation

$$\|f\|_{p(\cdot)} \leqslant C\left\|\sum \chi_Q \|f\chi_Q\|_{p(\cdot)}/\|\chi_Q\|_{p(\cdot)}\right\|_{p(\cdot)}.$$

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