

ON BOUNDS FOR THE SMALLEST AND THE LARGEST EIGENVALUES OF GCD AND LCM MATRICES

ERCAN ALTINIŞIK AND ŞERİFE BÜYÜKKÖSE

Abstract. In this paper, improving a famous result of Wolkowicz and Styan for the GCD matrix (S_n) and the LCM matrix $[S_n]$ defined on $S_n = \{1, 2, \dots, n\}$, we present new upper and lower bounds for the smallest and the largest eigenvalues of (S_n) and $[S_n]$ in terms of particular arithmetical functions.

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