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CHEBYSHEV AND GRÜSS TYPE INEQUALITIES INVOLVING TWO LINEAR FUNCTIONALS AND APPLICATIONS

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Abstract. In the present paper we prove the Chebyshev inequality involving two isotonic linear functionals. Namely, if A and B are isotonic linear functionals, then $A(pfg)B(q)+A(p)B(qfg) \geqslant A(pf)B(qg)+A(pg)B(qf)$, where p,q are non-negative weights and f,g are similarly ordered functions such that the above-mentioned terms are well-defined. If functionals are equal, i.e. $A=B$ and if $p=q$, then the above inequality becomes the Chebyshev inequality involving one isotonic linear functional: $A(p)A(pfg) \geqslant A(pf)A(pg)$ in which we recognize a generalization of the well-known classical integral and discrete Chebyshev inequalites as special cases.

We derive various properties of functionals related to the difference of the right-hand and the left-hand sides of the above-mentioned inequalities. The most remarkable results are the Grüss type inequalities for two functionals. Inequalities involving some fractional integral operators are also given.

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Paper MIA-19-10

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