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NOTES ON THE HERZ–TYPE HARDY SPACES OF VARIABLE SMOOTHNESS AND INTEGRABILITY

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Abstract. The aim of this paper is twofold. First we give a new norm equivalents of the variable Herz spaces $K^{\alpha(\cdot)}_{p(\cdot),q(\cdot)}(\mathbb{R}^n)$ and $\dot{K}^{\alpha(\cdot)}_{p(\cdot),q(\cdot)}(\mathbb{R}^n)$. Secondly we use these results to prove the atomic decomposition for Herz-type Hardy spaces of variable smoothness and integrability. Also, we prove the boundedness of a wide class of sublinear operators on these spaces, which includes maximal, potential and Calderón-Zygmund operators.

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