STRONG CONVERGENCE THEOREM
FOR WALSH–MARCINKIEWICZ MEANS

KÁROLY NAGY AND GEORGE TEPHNADZE

Abstract. It is known that the maximal operator of Walsh-Marcinkiewicz means is bounded from the dyadic martingale Hardy space H_p to the space L_p for $p > 2/3$ and the condition $p > 2/3$ is essential. In the case $p = 2/3$ the boundedness of the maximal operator does not hold. This means that the investigation of the maximal operator at the endpoint case $p = 2/3$ plays an important role.

The main aim of this paper is to prove a strong convergence theorem for the Walsh-Marcinkiewicz means on the Hardy space $H_{2/3}$.

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