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TRIANGLES, PARAMETERS, MODULUS OF SMOOTHNESS IN NORMED SPACES

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Abstract. Around 15 years ago several authors studied the parameter defined by

$$A_2(X) = \sup\left\{\frac{||x+y|| + ||x-y||}{2} : x,y \in S_X\right\},$$

where S_X denotes the unit sphere of the real Banach space X. In this paper we consider the new family of parameters that generalize $A_2(X)$:

$$A_{2,p}(X) = \sup\left\{\frac{||x+y|| + ||x-y||}{2} : x,y \in X,\ ||(||x||, ||y||)||_p \leqslant 2^{\frac{1}{p}}\right\}, 1 \leqslant p \leqslant \infty.$$

In this way, $A_{2,\infty}(X)$ is nothing else than $A_2(X)$ and we show how some interesting properties of real Banach spaces can be characterized by using our new constants.

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Paper MIA-19-15

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