POSITIVITY OF SUMS AND INTEGRALS FOR CONVEX FUNCTIONS OF HIGHER ORDER OF \( n \) VARIABLES

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Abstract. We provide one general discrete identity for \( \sum \cdots \sum P_{k_1 \cdots k_n} f(x_{k_1}, \ldots, x_{k_n}) \) and one general integral identity for \( \Lambda(f) = \int \cdots \int P(x_1, \ldots, x_n) f(x_1, \ldots, x_n) \, dx_1 \cdots dx_n \) of Popoviciu type. We obtain necessary and sufficient conditions under which these sum and integral are non-negative for higher order convex functions of \( n \) variables. These identities and inequalities generalize various established results. We also state new generalized Lagrange type and Cauchy type mean value theorems. We obtain an Ostrowski type result as a special case of our main integral identity and we also establish a bound on remainder term of our main integral identity in terms of \( L_p \)-norm by using Hölder’s inequality. Finally, we apply the functional \( \Lambda(f) \) on the family of some exponentially convex functions and discuss some of its major properties.


Keywords and phrases: Convex function of higher order, exponential-convexity.

References