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POSITIVITY OF SUMS AND INTEGRALS FOR CONVEX FUNCTIONS OF HIGHER ORDER OF n VARIABLES

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Abstract. We provide one general discrete identity for $\sum\cdots\sum P_{k_1\cdots k_n} f(x_{1k_1},\ldots,x_{nk_n})$ and one general integral identity for $\Lambda(f)=\int\cdots\int P(x_1,\ldots,x_n)f(x_1,\ldots,x_n)dx_1\cdots dx_n$ of Popoviciu type. We obtain necessary and sufficient conditions under which these sum and integral are non-negative for higher order convex functions of n variables. These identities and inequalities generalize various established results. We also state new generalized Lagrange type and Cauchy type mean value theorems. We obtain an Ostrowski type result as a special case of our main integral identity and we also establish a bound on remainder term of our main integral identity in terms of L_p-norm by using Hölder's inequality. Finally we apply the functional $\Lambda(f)$ on the family of some exponentially convex functions and discuss some of its major properties.

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