

A CLASS OF CONTINUED FRACTION INEQUALITIES

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Abstract. Given a finite sequence of positive real numbers, we construct terminating continued fractions whose partial denominators are formed by the arrangement of the numbers according to simple rules. This does not impose any restriction on the generality of our results and all simple continued fractions can be recast and formed according to these rules. After showing that the resulting finite continued fractions are multivariate convex or concave functions of the given sequence, we derive a class of inequalities using results from the theory of majorization. The main result of this paper is expressed in the form of inequalities connecting certain types of finite continued fractions and Fibonacci numbers.

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