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NEW ESTIMATES OF ESSENTIAL NORM OF AN INTEGRAL–TYPE OPERATOR BETWEEN BLOCH–TYPE SPACES ON THE UNIT BALL

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Abstract. Let $H(\mathbb{B})$ be the class of all holomorphic functions on the unit ball $\mathbb{B}$ in $\mathbb{C}^n$. For $g \in H(\mathbb{B})$ with $g(0) = 0$ and φ a holomorphic self-map of $\mathbb{B}$, the integral-type operator P^g_φ, recently introduced by S. Stević, is defined by

$$P^g_\varphi(f)(z) = \int_0^1 f(\varphi(tz))g(tz)\frac{dt}{t}, \quad z \in \mathbb{B}.$$

In this paper, we give a new characterization for the boundedness of integral-type operator P^g_φ between Bloch-type spaces on the unit ball $\mathbb{B}$. We also calculate the essential norm of the operator P^g_φ, which leads to a new condition for the compactness of the operator P^g_φ.

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